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LATERAL BUCKLING OF ROLLED STEEL BEAMS

R. A. Hechtman, A.M. ASCE,[1] J. S. Hattrup, J.M. ASCE,[2]
and J. L. Tiedemann

SYNOPSIS

In this experimental investigation of the lateral buckling of rolled steel beams under transverse loading, thirty-three beams with simple beam support conditions in depths from ten to eighteen inches and twenty-one ten-inch beams with two types of bolted semi-rigid end connections were tested. Good correlation between the experimental and the theoretical values of the nominal bending strength was found for the more slender simply supported beams which buckled in the elastic range. The factor of safety in terms of nominal bending strength of the test beams with respect to present and proposed design formulas was determined.

INTRODUCTION

Prior to 1946, the design of rolled steel beams with laterally unsupported length was related to the slenderness ratio, L/b.* Noting that this ratio neglects the influence of the torsional rigidity of the section and the restraint afforded by the tension flange, Karl deVries(1) extended the work of S. Timoshenko(2) and developed a new parameter, Ld/bt. The former expressed the critical load and nominal bending stress for a simply supported bisymmetrical I-shaped beam loaded in the plane of its web in essentially the following forms:

$$P_{cr} = \frac{EI_y d}{L^3} (16K), \tag{1}$$

$$f_{cr} = \frac{I_y}{I_x} \frac{d^2}{L^2} EK, \tag{2}$$

P_{cr} being the total load on the beam. The above equations are written for the loading condition of two equal concentrated loads at the quarter-points of the span length.

The parameter, Ld/bt, was deduced by deVries from the elastic buckling strength of a large number of rolled beams under uniform top-flange load computed by equations of the general form of Eq. 2. He developed the formula,

$$f_{cr} = \frac{20{,}000{,}000}{\frac{Ld}{bt}}, \tag{3}$$

*Definitions of the symbols are given at the end of this paper.

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as a reasonable basis for determining the buckling strengths for common types of transverse loading. Winter[3] showed that the buckling strength for such a beam under pure bending could be expressed by the equation,

$$f_{cr} = \frac{E\pi^2}{2\left(\frac{L}{d}\right)^2}\sqrt{\left(\frac{I_y}{2I_x}\right)^2 + \frac{KI_y}{2(1+v)I_x^2}\left(\frac{L}{\pi d}\right)^2}, \qquad (4)$$

and that with certain approximations, which were slight for almost all rolled beams, the parameter Ld/bt and Eq. 3 could be derived from this equation. The application of these same approximations to the quantity, $I_y d^2 k/I_x L^2$, in Eq. 2 will also result in this form of the slenderness ratio and Eq. 3.

For the design of slender beams, Karl deVries[1] incorporated a factor of safety of 1.67 in Eq. 3 and suggested the formula,

$$f = \frac{12{,}000{,}000}{\frac{Ld}{bt}}. \qquad (5)$$

Equation 5 was adopted by the American Institute of Steel Construction[4] in 1946 for the design of structural carbon steel rolled sections, plate girders, and built-up members in bending with an Ld/bt ratio in excess of 600. For shorter beams an allowable unit bending stress of 20,000 psi is used.

The L/b parameter for beams with laterally unsupported lengths is in use in other steel specifications.[15,16] A formula based on it may be found in Eq. 7.

An expression for the critical buckling load of beams under two equal symmetrically placed transverse loads was developed by Winter and Schrader:[5]

$$P_{cr} = \frac{AE}{L^3}\left[-\frac{dI_y}{2} + \sqrt{\left(\frac{dI_y}{2}\right)^2 + B\left(\frac{dI_y}{2}\right)^2 + DCI_yL^2}\right] \qquad (6)$$

The factors A, B, and D depend upon the same variables as the factor k in Eqs. 1 and 2.

The present investigation had as its principal objectives,

1) The testing of rolled steel beams of the wide-flange and standard I-beam types.
2) Correlation of the test results with theory where possible.
3) A critical examination of the Ld/bt parameter as a basis for predicting the buckling strength of standard rolled beams.
4) A preliminary exploration of the anticipated increase in strength that semi-rigid end connections would afford.
5) The determination of the factor of safety of rolled steel beams with respect to present and proposed design formulas.

Values of the factor k in Eq. 2 were available for two cases of interest in this investigation, simply supported beams with a midspan concentrated load or with uniform load over the entire span length.[1] There was the possibility of the existance of more severe loading conditions. Therefore, the methods by which Eqs. 1 and 2 were derived were employed[6] to compute the values of k for other loadings as shown in Fig. 1. It may be seen in this plot that the most severe loading of the types investigated was that of two

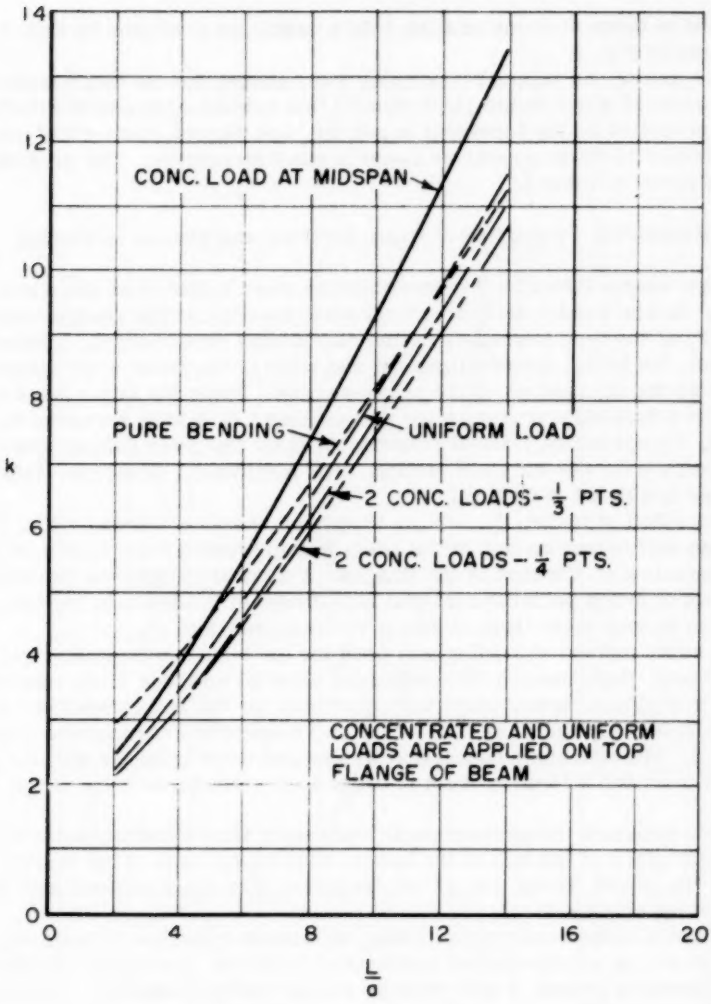


Fig. 1. Theoretical k-Values for Simply Supported Beams.

equal loads at the quarter-points of the beam. This loading was used for all the tests.

It had been previously pointed out[7,8] that the parameter Ld/bt would predict the value of the buckling strength less accurately for shorter beams than for longer since the shape of the cross-section is more significant. The desirability of choosing as test specimens three rolled sections with almost identical d/bt ratios, but with considerable difference in the shape of their cross-sections, led to the choice of the 10I25.4, 12WF27, and 18WF50 sections with d/bt ratios of 4.37, 4.60, and 4.22, respectively. These three sections, the first with tapered flanges and the remaining two with almost parallel flanges, would be expected to develop almost equal buckling strengths for the same span according to Eq. 3. The theoretical elastic buckling

strengths of these sections and the 10B15 beams as predicted by Eqs. 2 and 3 are shown in Fig. 2.

Two types of end support conditions were chosen for the test beams: first, simple support which would yield results that could be compared with the values predicted by the foregoing equations, and second, semi-rigid connections similar to those commonly found in steel structures. The program of tests is given in Table I.

Mechanical Properties of Beam Sections and Method of Testing

All the beams tested in this investigation were rolled from the same heat of plain-carbon semi-killed structural steel meeting ASTM Designation A 7-51T, or the type commonly called "structural carbon steel." Before each test, the initial out-of-alignment and twist of the beam were measured as well as the dimensions of the cross-section. From the latter were computed the actual slenderness ratios and moments of inertia recorded in Table I. Compression tests of coupons cut from the beam flanges gave the stress-strain curves for each section. The mechanical properties thus determined are listed in Table I.

The method of testing the simply supported beams is shown in Fig. 3. The span-end cross-section of the beam was prevented from tipping by lateral restraints on the toes of the flanges. A method of applying the loads was evolved which permitted manual adjustment of the loads in the lateral direction to keep their lines of action vertical (see Fig. 4).

The same method of loading was used for the beams with semi-rigid end connections. Rigid faces which remained parallel and free from rotation, but permitted axial translation, were provided for the end connections in the manner shown in Fig. 5. Details of the end connections themselves appear in Fig. 6. The web angle connection was scaled down from the standard beam connection[4] for a 14-inch depth in order to provide three bolts in line in the web.

In this program the quarter-point loads were first carefully located over the center plane of the web of the beam. Only in the case of the shorter Beams No. 25-30, 37-40, and 47-50, inclusive, was the additional procedure of centering employed.

In a beam under transverse loading, no simple criterion of centering exists as for an axially-loaded symmetrical column. When the initially curved beam is loaded, it will twist as well as deflect laterally. The centering criterion could then be either of two possibilities: placement of the load, first, so that no lateral deflection of the top flange of the beam at midspan occurred or, second, so that no rotation of the midspan cross-section developed. The latter criterion was chosen after the earlier tests showed that the rotations of the beam cross-sections were more sensitive to the positions of the loads and were more important in determining the buckling strength of the beam. Each load was adjusted laterally independently until the cross-sections under the loads and at midspan showed little or no rotation at a load estimated to be about one-fourth the buckling load for the beam. After centering, the test was begun.

The lateral deflections of the top and bottom flanges, the vertical deflections of the beam, and the rotations of the cross-sections were measured at the sixth-points of the span length at frequent load intervals up to buckling. The unit strains on the four toes of the beam flanges at midspan were determined by SR-4 strain gages. These data are not presented in this paper.

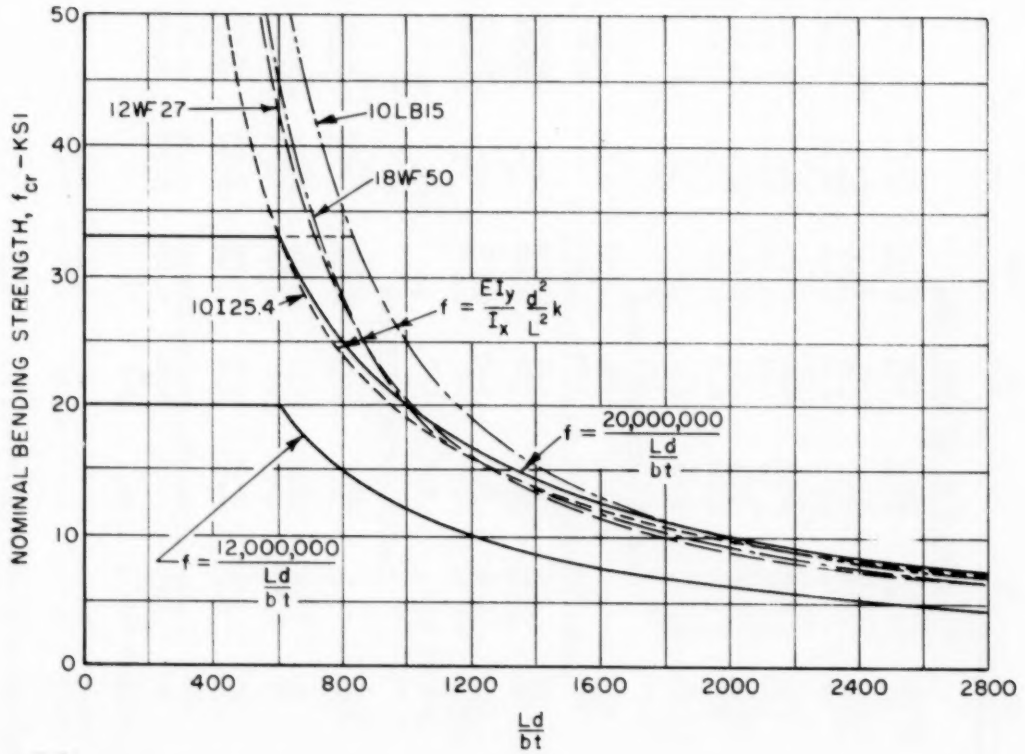


Fig. 2. Theoretical Nominal Bending Strength of Simply Supported Test Sections for Quarter-Point Loading.

TABLE I

PROPERTIES OF TEST BEAMS

Beam No.	Span	Slenderness Ratio, Ld/bt		Actual Moment of Inertia*		Average Compression Properties#		
		Actual*	Handbook	I_x	I_y	Upper Yield Point	Prop. Limit	Modulus of Elasticity
	In.			In.^{4}	In.^{4}	psi	psi	psi
Simply Supported 10I25.4 Beams								
1	120	576	526	119.9	6.37	35,500	30,200	31,600,000
2	120	574	526	115.8	6.36	35,600	30,500	29,200,000
3	150	696	658	125.0	6.83	33,500	31,700	31,600,000
4	150	700	658	116.5	6.54	34,500	30,000	31,100,000
5	240	1115	1052	125.0	6.83	33,500	31,700	31,600,000
6	240	1120	1052	116.5	6.54	34,500	30,000	31,100,000
7	360	1720	1576	114.5	6.36	35,600	30,500	29,200,000
8	360	1731	1576	118.5	6.37	35,500	30,200	31,600,000
		Handbook Value		122.1	6.9			
Simply Supported 10B15 Beams								
25	60	546	558	68.6	2.86	37,500	30,000	28,400,000
26	60	555	558	69.0	2.87	41,600	—	28,800,000
27	90	832	837	69.0	2.87	41,600	—	28,800,000
28	90	832	837	69.0	2.87	41,600	—	28,800,000
29	120	1111	1116	68.6	2.86	42,500	—	29,200,000
30	120	1111	1116	68.6	2.86	42,500	—	29,200,000
		Handbook Value		68.8	2.79			
Simply Supported 12WF27 Beams								
9	120	575	552	207.2	17.35	38,500	31,200	30,400,000
10	120	576	552	200.3	16.40	38,500	31,200	30,400,000
11	150	723	690	203.2	16.98	36,500	30,300	30,300,000
12	150	705	690	203.9	16.83	37,900	33,800	30,300,000
13	240	1130	1104	206.2	16.83	37,900	33,800	30,300,000
14	240	1155	1104	203.2	16.98	36,500	30,300	30,300,000
15	360	1725	1656	203.8	16.40	38,500	31,200	30,400,000
16	360	1680	1656	200.7	16.66	39,500	31,600	30,700,000
		Handbook Value		204.1	16.6			

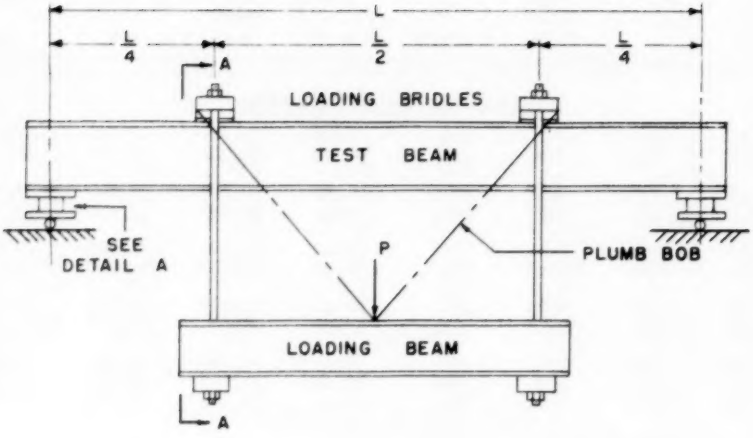
* Computed from measured dimensions of beams.

\# Compression properties are averages of results for two specimens from each mill section, one from each beam flange. 2-in. gage length.

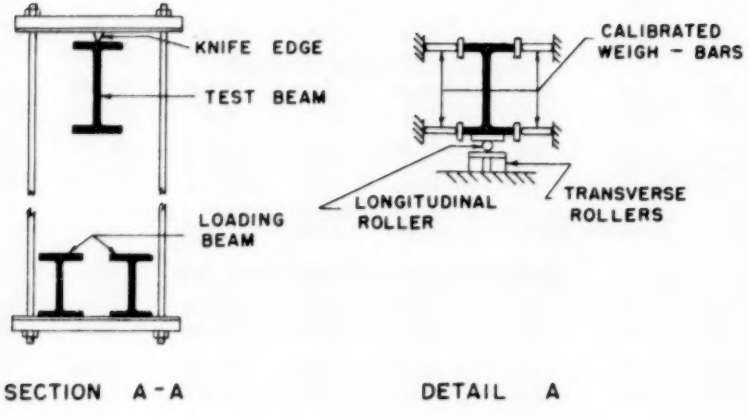
TABLE I (Cont.)

PROPERTIES OF TEST BEAMS

Beam No.	Span	Slenderness Ratio, Ld/bt		Actual Moment of Inertia*		Average Compression Properties#		
		Actual*	Handbook	I_x	I_y	Upper Yield Point	Prop. Limit	Modulus of Elasticity
	In.			In.^{4}	In.^{4}	psi	psi	psi
Simply Supported 18WF50 Beams								
17	120	530	506	799	37.42	39,200	34,000	29,900,000
18	120	534	506	764	36.04	38,300	34,200	28,700,000
19	150	664	633	799	37.42	39,200	34,000	29,900,000
20	150	666	633	764	36.04	38,300	34,200	28,700,000
21	240	1070	1012	799	37.42	39,200	34,000	29,900,000
22	240	1069	1012	764	36.04	38,300	34,200	28,700,000
23	360	1600	1518	789	36.42	39,800	36,200	30,100,000
24	360	1600	1518	785	35.73	38,500	36,000	29,800,000
55a	600	2660	2530	799	37.42	39,200	34,000	29,900,000
55b	600	2660	2530	799	37.42	39,200	34,000	29,900,000
56	600	2670	2530	764	36.04	38,300	34,200	28,700,000
		Handbook Value		800.6	37.2			
10B15 Beams With Top and Seat Angle Connections								
37	90	822	837	70.5	2.96	43,800	39,500	29,800,000
38	90	822	837	70.5	2.96	43,800	39,500	29,800,000
39	120	1111	1116	67.6	2.79	43,300	40,500	27,900,000
40	120	1111	1116	67.6	2.79	43,300	40,500	27,900,000
41	180	1679	1674	68.4	2.82	38,300	37,000	27,700,000
42	180	1679	1674	68.4	2.82	38,300	37,000	27,700,000
43	240	2207	2232	68.8	2.90	40,500	35,500	28,600,000
44	240	2215	2232	68.5	2.87	40,200	36,000	28,700,000
57a	360	3290	3348	70.5	2.96	43,800	39,500	29,800,000
57b	360	3290	3348	70.5	2.96	43,800	39,500	29,800,000
58	360	3350	3348	67.6	2.79	43,300	40,500	27,900,000
		Handbook Value		68.8	2.79			
10B15 Beams With Web Angle Connections								
47	90	826	837	68.1	2.82	42,600	40,200	27,800,000
48	90	826	837	68.1	2.82	42,600	40,200	27,800,000
49	120	1108	1116	68.1	2.82	41,500	33,000	26,600,000
50	120	1108	1116	68.1	2.82	41,500	33,000	26,600,000
51	180	1649	1674	68.5	2.86	41,700	31,000	28,300,000
52	180	1649	1674	68.5	2.86	41,700	31,000	28,300,000
53	240	2268	2232	67.9	2.90	40,300	35,700	29,700,000
54	240	2183	2232	68.6	2.86	37,500	30,000	28,400,000
59	360	3302	3348	68.1	2.82	42,600	40,200	27,800,000
60	360	3320	3348	68.8	2.82	41,500	33,000	26,600,000
		Handbook Value		68.8	2.79			



TEST BEAM AND LOADING BEAM



SECTION A-A DETAIL A

Fig. 3. End Supports and Method of Loading for Simply Supported Beams.

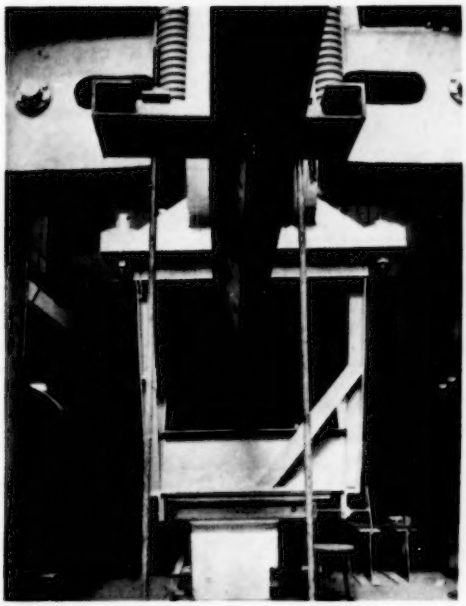


Fig. 4. Fixture for Adjusting Load Laterally.

However, they served as a very useful means of observing the behavior of the beams and the iminence of failure.

Results of Tests

Summaries of the more important data of these tests appear in Tables II and III.

The plot in Fig. 8 shows the relation between the nominal bending strength and the actual slenderness ratio Ld/bt of the simply supported beams. This figure and the individual plots in Figs. 9 and 10 indicate that the buckling strengths of the shorter beams of the four section sizes fell more or less in the same order as the theoretical strengths plotted in Fig. 2., the 10B15 section having the highest strength, the 10I25.4 the lowest, and the 12WF27 and the 18WF50 falling in an intermediate position.

It seems reasonable to conclude that the Ld/bt parameter should be adequate for designing the wide-flange and standard beams in the Manual of the American Institute of Steel Construction.[4] The superiority of this parameter over L/b may be seen by examining the results of these tests when plotted against the latter in Fig. 11. Here there is little tendency for the data to fall along a common curve. The reason for the failure of the L/b parameter to predict the buckling strengths of the beams can be explained. It can be shown that the transformation of the quantity, $I_y d / I_x L$, in Eq. 2 in terms of b, t, d, and L will result in the L/b parameter. However, the important factor k is then omitted from the parameter. The parameter Ld/bt contains the quantity k.

The nominal bending strength developed by the beams with the top-and-seat-angle and the web-angle connections was computed on the assumption

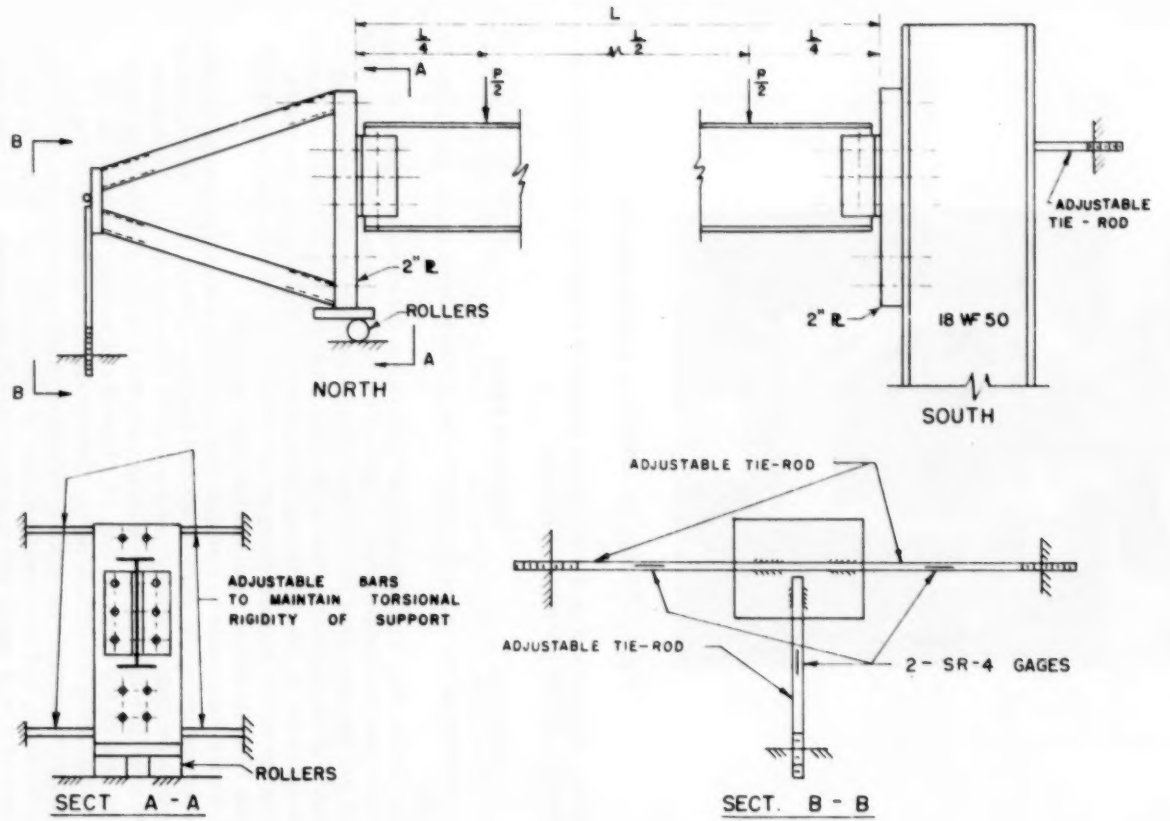
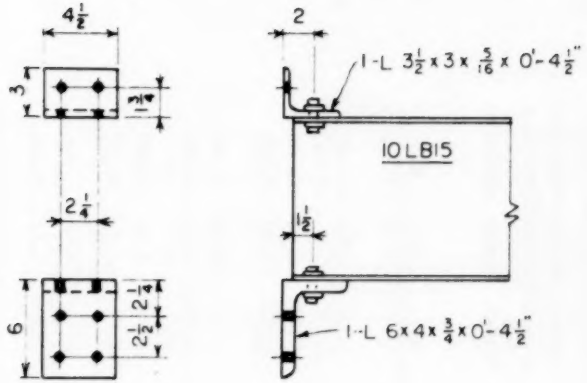
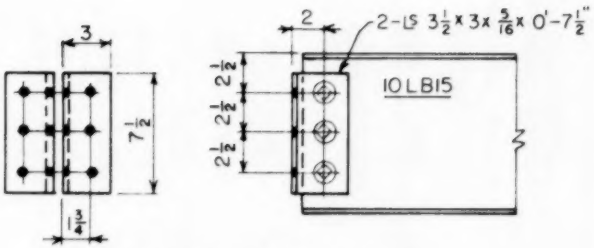


Fig. 5. End Supports and Method of Loading. Beams with Semi-Rigid End Connections.



⅝" ϕ HIGH TENSILE STEEL BOLTS
ALL HOLES DRILLED 11/16" ϕ
CARBURIZED WASHER UNDER
HEAD AND NUT OF BOLT

TOP AND SEAT ANGLE CONNECTION



WEB ANGLE CONNECTION

Fig. 6. Details of Semi-Rigid End Connections.

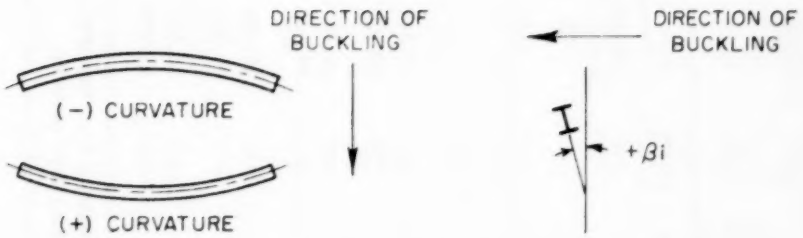


Fig. 7. Sign Convention for Initial Misalignment and Twist.

TABLE II

SUMMARY OF DATA FOR TESTS OF SIMPLY SUPPORTED BEAMS

Beam No.	Actual Ld/bt*	Nominal Bending Strength at Critical Load		Critical Buckling Load					Initial Out-of-Alignment of Flanges at Midspan#		
		20,000,000* Ld/bt	Test Value	Test Value**	Southwell's Method		Theoretical Values		Top Flange	Bottom Flange	Angle of Twist
					Donnell	Lundquist	Modified de Vries Eq. 2	Winter-Schrader Eq. 6			
		psi	psi	kips	kips	kips	kips	kips	In.	In.	Rad.
				10I25.4 Sections							
1	576	33,000	27,000	44.1	—	54.0	58.0	63.0	-0.03	-0.07	0.004
2	574	33,000	25,300	41.4	—	—	58.0	60.0	-0.04	-0.08	0.003
3	696	28,800	22,900	29.9	—	40.0	36.0	41.0	0.12	-0.02	0.014
4	700	28,600	21,800	28.5	—	32.0	36.4	39.0	0.08	0.02	0.005
5	1115	18,000	16,000	13.1	13.5	13.1	13.8	15.7	0.10	0.03	0.007
6	1120	17,900	17,400	14.2	14.3	14.2	13.8	14.8	-0.02	-0.09	0.007
7	1720	11,600	12,600	6.8	6.8	6.6	6.2	6.4	0.11	0.08	0.003
8	1731	11,600	11,600	6.3	6.9	6.6	6.3	6.7	0.19	0.13	0.006

* Computed from measured dimensions of beams.

** Total load on beam.

\# Out-of-alignment with respect to span end points. See Fig. 7 for sign convention.

TABLE II (Cont.)

SUMMARY OF DATA FOR TESTS OF SIMPLY SUPPORTED BEAMS

Beam No.	Actual Ld/bt*	Nominal Bending Strength at Critical Load		Critical Buckling Load					Initial Out-of Alignment of Flanges at Midspan#		
		20,000,000* Ld/bt	Test Value	Test Value**	Southwell's Method		Theoretical Values		Top Flange	Bottom Flange	Angle of Twist
					Donnell	Lundquist	Modified de Vries Eq. 2	Winter-Schrader Eq. 6			
		psi	psi	kips	kips	kips	kips	kips	In.	In.	Rad.
				10B15 Sections							
25	546	33,000	34,100	62.7	80.0	104.0	111.0	122.0	0.01	-0.01	0.002
26	555	33,000	33,600	61.7	80.0	107.0	114.0	125.0	-0.02	-0.03	0.001
27	832	24,000	30,600	37.6	44.3	40.9	40.6	44.4	-0.01	0	-0.001
28	832	24,000	28,600	35.1	41.3	41.6	40.6	44.4	0.01	0.02	-0.001
29	1111	18,000	21,100	19.4	21.7	21.2	18.9	21.1	0.04	-0.06	0.010
30	1111	18,000	21,200	19.5	21.4	20.9	18.9	21.1	0.04	-0.03	0.007
				12WF27 Sections							
9	575	33,000	29,500	66.9	—	114.0	108.0	121.0	0.01	-0.04	0.004
10	576	33,000	28,400	64.3	—	70.0	108.0	116.0	0	0	0
11	723	27,700	25,700	46.9	—	—	63.0	67.0	-0.01	0.06	-0.006
12	705	28,400	20,300	36.9	—	—	60.0	64.0	0.02	0	0.002
13	1130	17,700	17,400	19.6	19.8	19.8	18.9	20.6	0.04	0.10	-0.005
14	1155	17,300	16,400	18.5	19.3	20.7	20.0	21.8	0.06	-0.11	0.014
15	1725	11,600	11,100	8.4	8.5	8.8	8.4	8.8	0.09	0.07	0.002
16	1680	11,900	11,500	8.7	8.8	9.0	8.3	9.0	0.12	0.12	0

TABLE II (Cont.)

SUMMARY OF DATA FOR TESTS OF SIMPLY SUPPORTED BEAMS

Beam No.	Actual Ld/bt*	Nominal Bending Strength at Critical Load		Critical Buckling Load					Initial Out-of-Alignment of Flanges at Midspan#		
		20,000,000* Ld/bt	Test Value	Test Value**	Southwell's Method		Theoretical Values		Top Flange	Bottom Flange	Angle of Twist
					Donnell	Lundquist	Modified de Vries Eq. 2	Winter-Schrader Eq. 6			
		psi	psi	kips	kips	kips	kips	kips	In.	In.	Rad.
				18WF50 Sections							
17	530	33,000	25,900	153.7	—	—	364.0	360.0	-0.03	-0.06	0.001
18	534	33,000	29,700	176.1	—	—	354.0	344.0	0	0	0
19	664	30,100	31,000	147.1	—	—	195.0	196.0	0.02	-0.06	0.004
20	666	30,100	26,200	124.2	—	163.0	195.0	190.0	0	-0.01	0.001
21	1070	18,700	22,800	67.6	73.0	72.0	58.0	60.0	0.02	0.12	-0.006
22	1069	18,700	21,500	63.8	66.0	70.0	60.0	60.0	0.02	0.01	0.001
23	1600	12,500	14,100	27.7	27.8	28.5	24.0	25.4	-0.10	-0.37	0.015
24	1600	12,500	12,200	24.0	25.2	25.4	23.6	24.8	0.10	0.07	0.002
55a	2660	7,500	6,880	8.2	8.4	8.4	7.7	8.0	0.08	0.01	0.004
55b	2660	7,500	7,000	8.3	8.5	8.5	7.7	8.0	-0.01	-0.08	0.004
56	2670	7,500	7,440	8.8	8.8	8.9	8.6	8.4	-0.02	-0.08	0.003

TABLE III

SUMMARY OF DATA FOR TESTS OF BEAMS WITH SEMI-RIGID END CONNECTIONS

Beam No.	Actual Ld/bt*	Nominal Bending Strength at Critical Load		Critical Buckling Load					Moment at Design Load***		Moment at Critical Load	
		20,000,000* Ld/bt	Test Value	Test Value**	Southwell's Method		Theoretical Values		M_x	M_y	M_x	M_y
					Donnell	Lundquist	Modified de Vries Eq. 2	Winter-Schrader Eq. 6				
		psi	psi	kips	kips	kips	kips	kips	kip-ft	kip-ft	kip-ft	kip-ft
				10LB15 Sections with Top and Seat Angle Connections								
37	822	24,300	35,600	43.7	54.0	53.5	40.0	42.0	4.60	-0.07	10.70	-0.01
38	822	24,300	38,100	46.7	50.5	54.1	40.0	42.0	3.82	-0.06	10.08	—
39	1111	18,000	35,500	32.7	—	33.6	18.7	20.0	3.12	-0.02	10.05	0.14
40	1111	18,000	29,700	27.3	30.8	31.0	18.7	20.0	2.72	-0.12	8.60	0.00
41	1679	11,900	21,800	13.4	14.1	14.4	7.6	7.9	2.26	0.00	7.59	0.35
42	1679	11,900	23,800	14.7	15.0	14.9	7.6	7.9	1.90	0.00	7.59	0.21
43	2207	9,000	19,800	9.1	—	9.5	4.5	4.7	1.66	0.03	7.56	0.30
44	2215	9,000	20,500	9.4	—	—	4.5	4.7	1.67	-0.09	7.75	-0.15
57a	3290	6,080	17,200	—	5.3	5.0	2.0	2.0	0.44	0.00	10.54	0.69
57b	3290	6,080	15,900	—	4.9	4.7	2.0	2.0	0.62	0.01	9.50	0.68
58	3350	5,970	15,000	—	4.6	4.5	1.9	2.1	0.61	0.02	9.05	0.73

* Computed from measured dimensions of beams.

** Total load on beam.

*** Design load computed on assumption of simple-beam support conditions. Values at design load found by interpolation between nearest experimental values.

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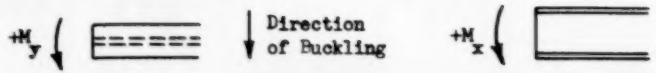

TABLE III (Cont.)

SUMMARY OF DATA FOR TESTS OF BEAMS WITH SEMI-RIGID END CONNECTIONS

Beam No.	Actual Ld/bt*	Nominal Bending Strength at Critical Load		Critical Buckling Load					Moment at Design Load***		Moment at Critical Load	
		20,000,000* Ld/bt	Test Value	Test Value**	Southwell's Method		Theoretical Values		M_x	M_y	M_x	M_y
					Donnell	Lundquist	Modified de Vries Eq. 2	Winter-Schrader Eq. 6				
		psi	psi	kips	kips	kips	kips	kips	kip-ft	kip-ft	kip-ft	kip-ft
				10LB15 Sections with Web Angle Connections								
47	826	24,200	34,800	42.7	45.5	45.2	40.0	41.2	3.03	0.02	7.00	0.14
48	836	24,200	33,200	40.7	44.5	44.7	40.0	41.2	3.00	-0.04	6.96	-0.05
49	1108	18,100	25,500	23.5	25.2	24.8	19.3	19.6	3.04	0.07	5.70	0.19
50	1108	18,100	27,200	25.0	25.2	25.1	19.3	19.6	2.75	0.00	6.30	0.05
51	1649	12,100	17,900	11.0	11.4	11.3	8.2	8.6	2.15	0.02	5.97	0.10
52	1649	12,100	18,100	11.2	11.6	11.5	8.2	8.6	1.73	0.00	4.36	0.17
53	2268	8,820	13,600	6.2	6.2	6.2	4.3	4.7	1.51	-0.01	5.53	0.67
54	2183	9,170	14,000	6.4	6.5	6.4	4.5	4.7	1.47	0.01	3.58	0.37
59	3302	6,070	12,400	3.8	3.8	3.7	2.0	2.0	0.54	0.02	5.41	0.53
60	3320	6,030	11,700	3.6	3.6	3.7	2.0	2.0	0.57	0.00	5.02	0.45

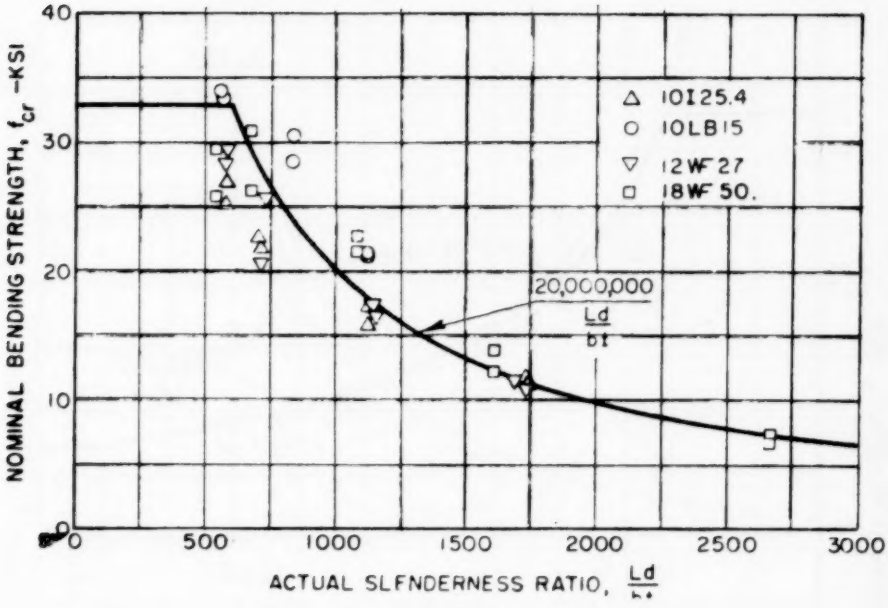

Fig. 8. Comparison of Nominal Bending Strength of Simply Supported Test Beams and Actual Slenderness Ratio Ld/bt.

that they were simply supported and is shown in Fig. 12. The actual nominal bending strength would be related to the effective rigidity of the end connections and was considerably less than the values shown in this figure. The greater rigidity developed by the top and seat angle connection as compared with the web angle connection is in line with previous observations.[9] It should be borne in mind that when identical semi-rigid connections are used for different span lengths of the same beam section, the effective end restraint will increase as the span length increases.[9]

In Fig. 13, the experimental or actual nominal bending strength of all the beams is compared with the theoretical value for the quarter-point loading. The points for Ld/bt ratios less than 600 were based upon a theoretical critical stress which was greater than the yield point of the steel and therefore must be so interpreted.

From Fig. 13 it is possible to determine the Ld/bt ratio below which the beams buckled in the inelastic range by observing the value at which there was a downward turn in the curves. These values were about 1100 for the simply supported beams and those with web angle connections and about 1300 for those with the top-and-seat-angle connections. Wherever the beams buckled in the elastic range, fairly good correlation was found for the simply supported beams between the experimental results and the predictions of theory. This correlation with theory would suggest similar agreement if types of loading other than quarter-point loading had been used in these tests.

The parameter L/a is used by Timoshenko[2] to describe the slenderness of a beam subject to lateral buckling. By means of Eq. 1 it was possible to

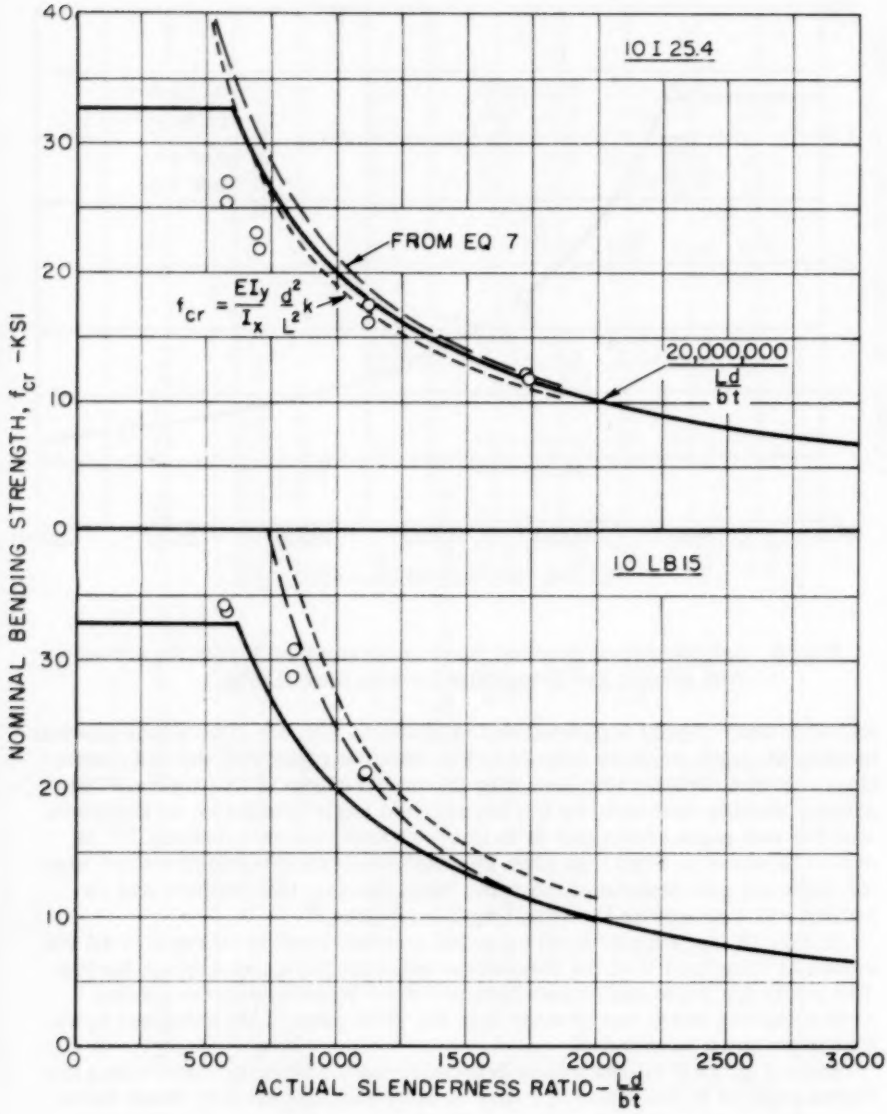

Fig. 9. Nominal Bending Strength of 10I25.4 and 10LB15 Beams.

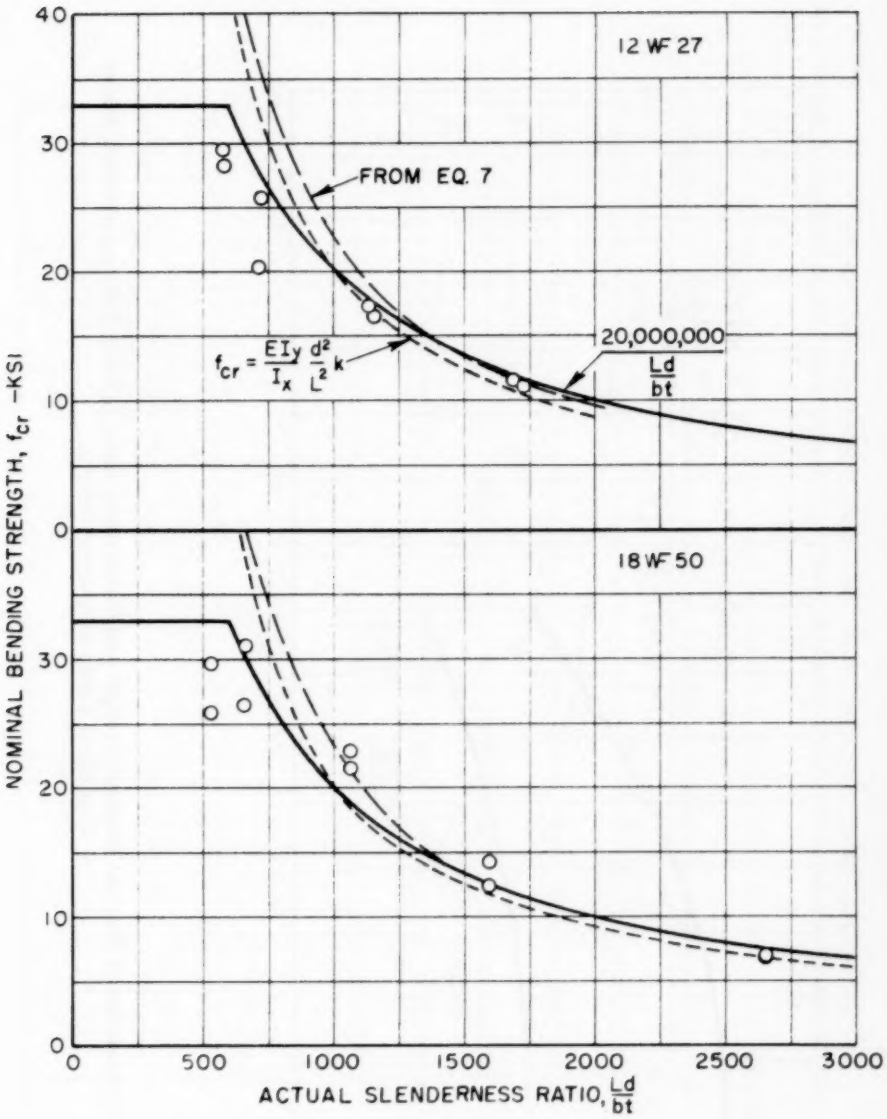

Fig. 10. Nominal Bending Strength of 12WF27 and 18WF50 Beams.

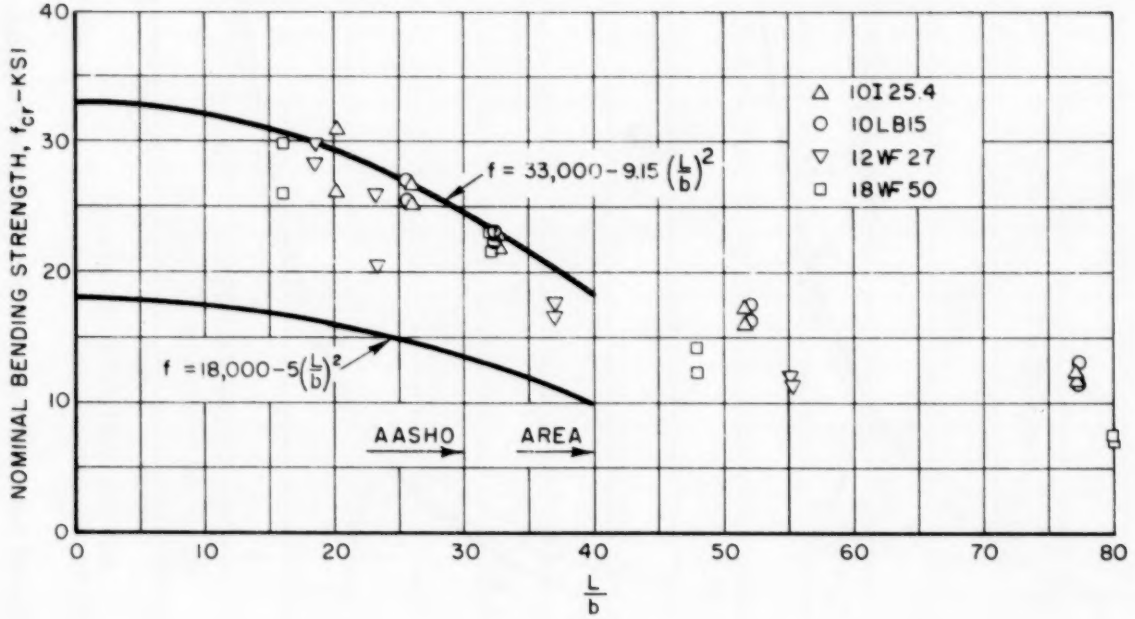

Fig. 11. Comparison of Nominal Bending Strength and Slenderness Ratio L/b for Simply Supported Beams. Two Equal Loads at Quarter-Points.

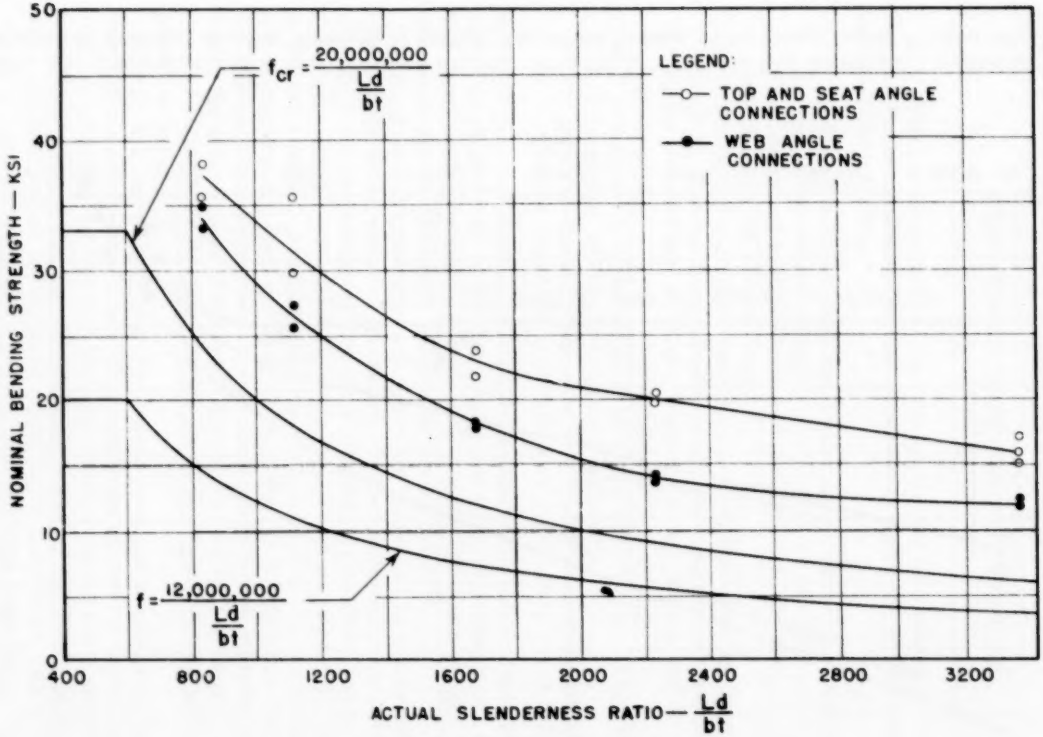

Fig. 12. Comparison of Nominal Bending Strength and Actual Slenderness Ratio Ld/bt for 10LB15 Beams with Bolted Semi-Rigid End Connections. Two Equal Loads at Quarter-Points.

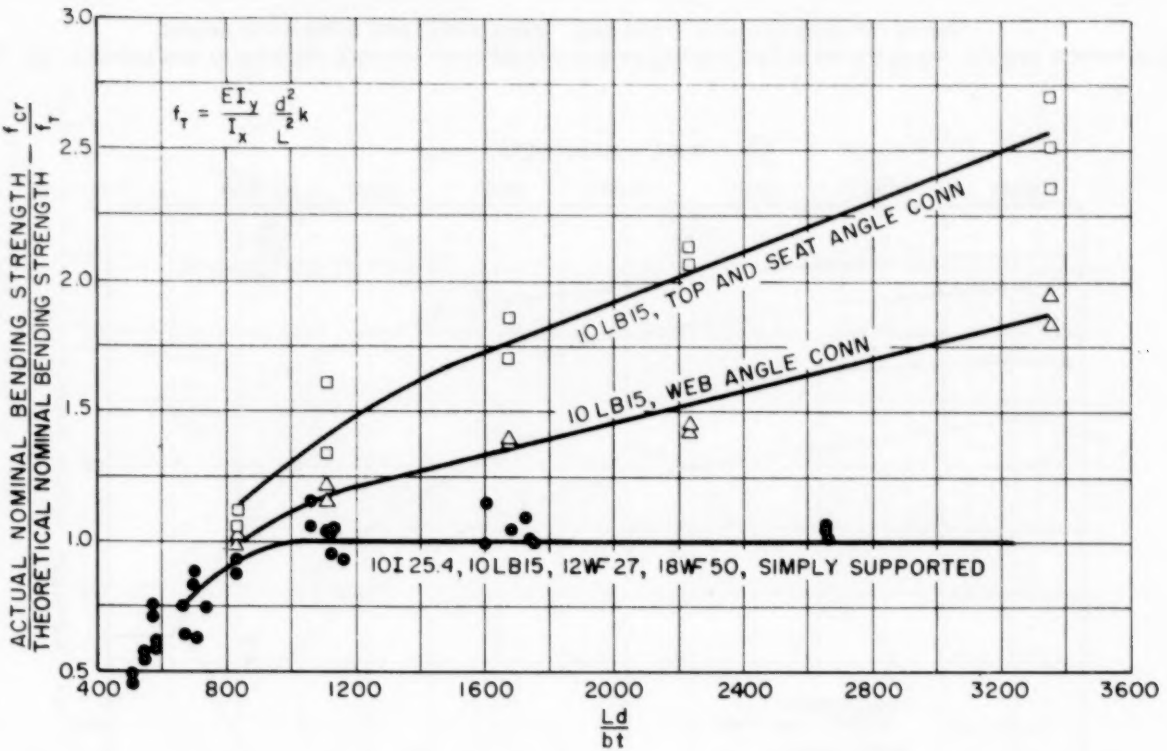

Fig. 13. Comparison of Actual Nominal Bending Strength for Various End Restraint Conditions with Theoretical Nominal Bending Strength of Simply Supported Beams. Two Equal Loads at Quarter-Points.

compute the factor k developed by the beams. These values are compared with the actual L/a value of the beams in Fig. 14. Where L/a was greater than about three, the simply supported beams failed in the elastic range and the values of k fell near the theoretical curve. Because of the varying span-end rigidity of the beams with semi-rigid connections, the experimental curves for these can best serve to give an approximate indication of the probable effectiveness of other similar connections.

Effect of Initial Misalignment and Centering on the Bending Strength

The initial misalignment of the test beams and its relation to the direction in which the beam finally buckled are given in Table II. These observations indicated the following:

1) The direction of buckling was always in the direction of the initial angle of twist of the web at midspan except in those cases where both flanges were deformed in the direction of buckling with the greater amount of curvature in the bottom flange.
2) The nature of the initial curvature and twist affected the buckling strength with the most adverse condition occurring when both the twist of the web and the curvature of the top flange were in the direction of buckling.
3) It was an asset to the buckling strength when the top flange was initially deformed against the direction of buckling.
4) The effect of misalignment upon the buckling strength of the simply supported beams appeared to decrease as the value of Ld/bt increased.

Since the misalignment of the top flange determined the position of the load, hence its eccentricity to the line joining the support points, it can be seen that these findings were quite logical. An exception to the pattern in (1) above was Beam No. 11 whose top flange was almost straight.

The purpose of centering is to move the load so as to counterbalance the effect of the initial shape of the beam. In (1) and (2) above the initial rotation of the cross-sections of the beam would be in the direction of buckling. Centering increased the load-carrying capacity of these beams. The beams in (3) above tended to twist during the early stages of the test in the direction opposite to the final direction of buckling. It is important that the placement of the loads during centering be determined by the direction of the twist during the later stages of the test. Therefore, the load used in centering must be sufficiently large to develop the mode of deformation which is to prevail at failure.

In Fig. 8 it can be seen that there were sometimes considerable differences in the strengths of some of the duplicate beams for the shorter 10I25.4, 12WF27, and 18WF50 sections which were not centered and, in contrast, better agreement for the duplicate beams of the 10B15 section which were centered. Thus the effect of centering in these tests was to produce more consistent results and a higher load-carrying capacity.

Southwell's Method of Determining Buckling Loads

In 1932 Southwell[12] described a method which employs an analysis of the applied loads and the resulting deflections observed in the test of an initially curved column to compute the buckling load which would occur if the column

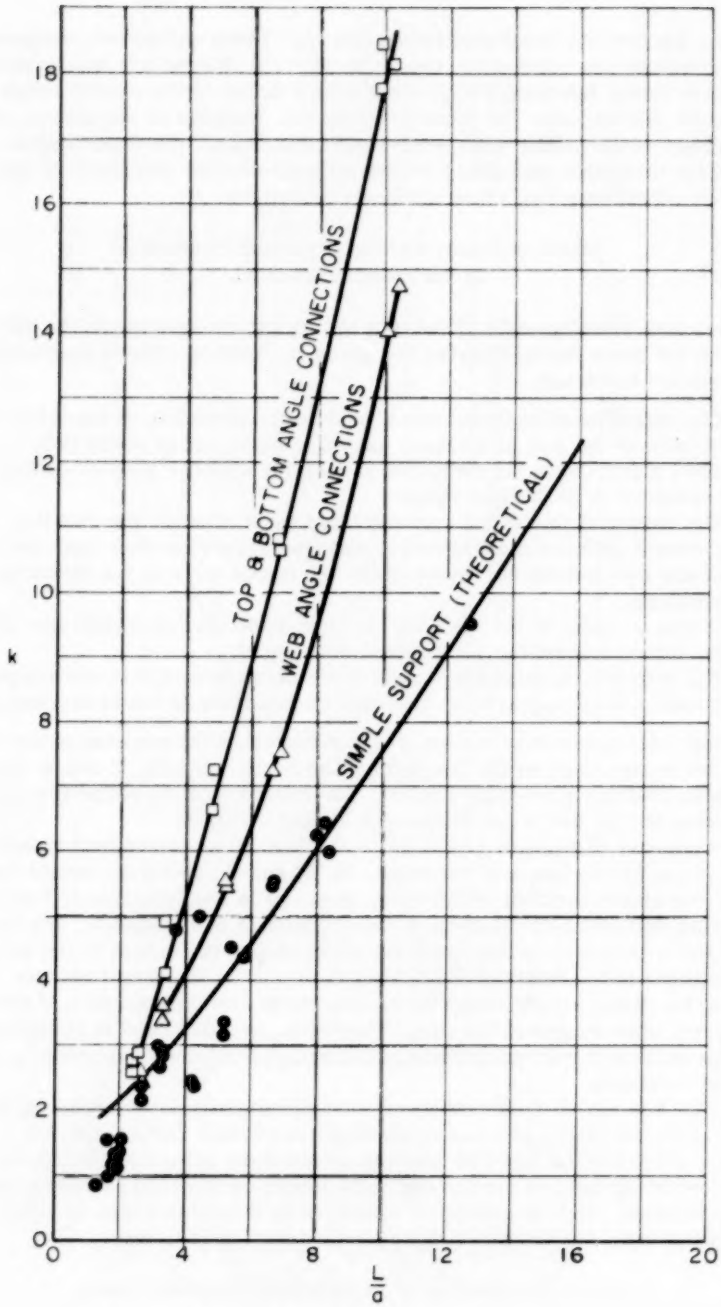

Fig. 14. Factor k Developed by Test Beams.

were perfectly straight. Variations of this method were later developed by Donnell[13] and Lundquist,[14] both of which were applied to these test data to yield the results given in Tables II and III. Where no value is shown, the plot produced no conclusive evidence. The superiority of Lundquist's variation over that of Donnell in organizing the data into a consistent pattern was indicated in these tests, especially for the shorter beams.

A typical plot for both methods is shown in Fig. 15. Since the rotation β of the cross-sections of a beam appears in the differential equations of equilibrium in a manner similar to that of the lateral deflection of a column, the angle of twist β was used in the Lundquist plots. This type of plot for the shorter beams consisted of three parts. At low values of the load, the curve shown by the dashed line in Fig. 15 was obtained. This was followed by a straight line, the inverse slope of which was used to compute the elastic buckling load. If the beam yielded, the last part of the plot developed a curve with increasing slope, the inverse slope of the tangent to this last part of the curve at the point of buckling was the actual critical load for the beam. That Southwell's plot would give this shape of curve was mentioned by Donnell.[13]

Southwell's method as an experimental tool could be said to have had three applications in these tests: first, for these beams which failed by yielding, it gave an approximate probable elastic buckling load as well as an indication of the actual buckling load; second, it could check the buckling load of the beams which failed in the elastic range; and third, it enabled the experimental method of loading and the restraint of the end connections to be checked.

An example of the last might be mentioned. The heel of the top connection angles of Beams No. 57a, 57b, and 58 came into bearing at one end on the face of the supporting columns because of the large lateral deflections of the top flange close to failure, and these beams never actually buckled because of this increased degree of restraint. The Lundquist plot for those beams near failure showed a decreasing slope corresponding to an apparent increase in buckling strength. By extrapolation of the straight-line portion of the plot prior to this occurrence, the probable buckling load for the initial degree of restraint was determined. This value was used to compute the nominal bending strength of these three beams.

Factor of Safety with Respect to Present and Proposed Design Formulas

The nominal bending strength developed by the test beams is compared in Table IV with the allowable unit bending stress permitted by the following present and proposed design formulas for structural carbon steel meeting ASTM Designation A 7-51T. These formulas were originally derived on the assumption that the beam was simply supported.

Formula in specifications of American Railway Engineering Association[15] and American Association of State Highway Officials:[16]

$$\begin{matrix}\text{For } L/b \leqq 40 \text{ (AREA)} \\ \text{For } L/b \leqq 30 \text{ (AASHO)}\end{matrix} \qquad f = 18{,}000 - 5(L/b)^2 \qquad (7)$$

Formula in specification of American Institute of Steel Construction:[4]

$$\text{For } Ld/bt \leqq 600 \qquad f = 20{,}000$$

$$\text{For } Ld/bt > 600 \qquad f = \frac{12{,}000{,}000}{\frac{Ld}{bt}} \qquad (8)$$

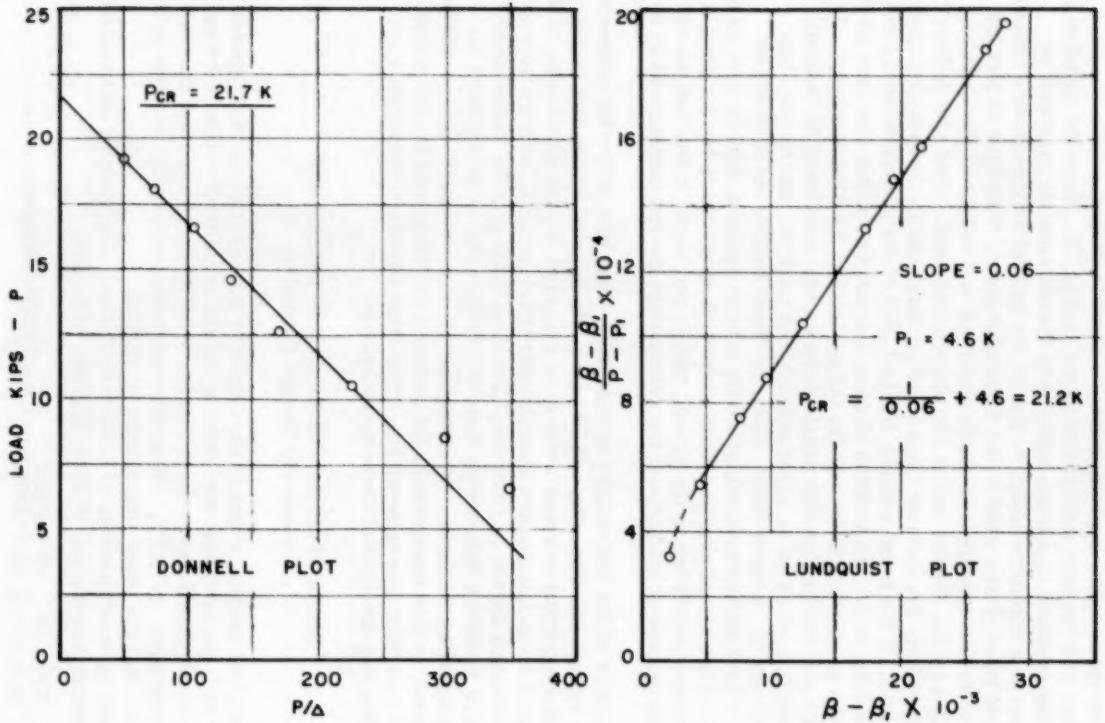

Fig. 15. Comparison of Lundquist and Donnell Plots to Determine Buckling Load for Beam No. 29.

TABLE IV

FACTOR OF SAFETY DEVELOPED BY BEAMS WITH RESPECT TO PRESENT AND PROPOSED DESIGN FORMULAS

Beam No.	Slenderness Ratio Ld/bt	Slenderness Ratio L/b	Factor of Safety#				
			20,000 psi Base, AISC	18,000 psi Base			
				AREA	AASHO	Proposed Formula	
						Parabolic Cutoff	Straight-Line Cutoff
			Simply Supported 10I25.4 Beams				
1	576	25.8	1.35	1.84	1.84	1.69	1.76
2	574	25.8	1.27	1.72	1.72	1.58	1.65
3	696	32.2	1.32	1.78	*	1.53	1.59
4	700	32.2	1.26	1.69	*	1.45	1.57
5	1115	51.6	1.48	*	*	1.48	1.45
6	1120	51.6	1.62	*	*	1.62	1.58
7	1720	77.3	1.81	*	*	1.81	1.81
8	1731	77.3	1.67	*	*	1.67	1.67
			Simply Supported 10B15 Beams				
25	546	15.0	1.71	2.02	2.02	2.10	2.18
26	555	15.0	1.68	1.99	1.99	2.07	2.16
27	832	22.5	2.13	1.99	1.99	2.22	2.30
28	832	22.5	1.98	1.86	1.86	2.07	2.15
29	1111	30.0	1.96	1.56	1.56	1.96	1.91
30	1111	30.0	1.96	1.57	1.57	1.96	1.92
			Simply Supported 12WF27 Beams				
9	575	18.4	1.48	1.81	1.81	1.84	1.92
10	576	18.4	1.42	1.74	1.74	1.78	1.85
11	723	23.1	1.55	1.68	1.68	1.72	1.82
12	705	23.1	1.19	1.33	1.33	1.35	1.42
13	1130	36.9	1.64	1.55	*	1.64	1.60
14	1155	36.9	1.58	1.46	*	1.58	1.52
15	1725	55.3	1.60	*	*	1.60	1.60
16	1680	55.3	1.61	*	*	1.61	1.61

Design formulas are as follows:
AISC, Eq. 8.
AREA and AASHO, Eq. 7.
' Proposed formulas: with parabolic cutoff, Eq. 9; with straight-line cutoff, Eq. 10.

* Slenderness ratio greater than that permitted by code.

TABLE IV (Cont.)

FACTOR OF SAFETY DEVELOPED BY BEAMS WITH RESPECT TO PRESENT AND PROPOSED DESIGN FORMULAS

Beam No.	Slenderness Ratio Ld/bt	Slenderness Ratio L/b	Factor of Safety#				
			20,000 psi Base, AISC	18,000 psi Base			
				AREA	AASHO	Proposed Formula	
						Parabolic Cutoff	Straight-Line Cutoff
Simply Supported 18WF50 Beams							
17	530	16.0	1.30	1.55	1.55	1.59	1.64
18	534	16.0	1.49	1.78	1.78	1.82	1.88
19	664	20.0	1.71	1.94	1.94	2.03	2.11
20	666	20.0	1.45	1.64	1.64	1.71	1.78
21	1070	32.0	2.03	1.78	*	2.03	2.00
22	1069	32.0	1.92	1.68	*	1.92	1.89
23	1600	48.0	1.88	*	*	1.88	1.88
24	1600	48.0	1.63	*	*	1.63	1.63
55a	2660	80.0	1.53	*	*	1.53	1.53
55b	2660	80.0	1.56	*	*	1.56	1.56
56	2670	80.0	1.66	*	*	1.66	1.66
10B15 Beams with Top and Seat Angle Connections							
37	822	22.5	2.44	2.31	2.31	2.56	2.66
38	822	22.5	2.61	2.47	2.47	2.74	2.84
39	1111	30.0	3.29	2.63	2.63	3.29	3.21
40	1111	30.0	2.75	2.20	2.20	2.75	2.68
41	1679	45.0	3.05	*	*	3.05	3.05
42	1679	45.0	3.33	*	*	3.33	3.33
43	2207	60.0	3.67	*	*	3.67	3.67
44	2215	60.0	3.80	*	*	3.80	3.80
57a	3290	90.0	4.72	*	*	4.72	4.72
57b	3290	90.0	4.37	*	*	4.37	4.37
58	3350	90.0	4.19	*	*	4.19	4.19
10B15 Beams with Web Angle Connections							
47	826	22.5	2.40	2.26	2.26	2.50	2.60
48	826	22.5	2.29	2.16	2.16	2.39	2.48
49	1108	30.0	2.35	1.89	1.89	2.35	2.30
50	1108	30.0	2.50	2.02	2.02	2.50	2.46
51	1649	45.0	2.47	*	*	2.47	2.47
52	1649	45.0	2.49	*	*	2.49	2.49
53	2268	60.0	2.57	*	*	2.57	2.57
54	2183	60.0	2.55	*	*	2.55	2.55
59	3302	90.0	3.41	*	*	3.41	3.41
60	3320	90.0	3.24	*	*	3.24	3.24

Formula proposed by Karl deVries[1] incorporating parabolic cutoff:

$$\begin{aligned} &\text{For } Ld/bt \leq 1000 && f = 18{,}000 - 0.006(Ld/bt)^2 \\ &\text{For } Ld/bt > 1000 && f = \frac{12{,}000{,}000}{\frac{Ld}{bt}} \end{aligned} \qquad (9)$$

Formula (17) incorporating diagonal straight-line cutoff:

$$\begin{aligned} &\text{For } Ld/bt \leq 250 && f = 18{,}000 \\ &\text{For } 250 < Ld/bt \leq 1500 && f = 20{,}000 - 8(Ld/bt) \\ &\text{For } Ld/bt > 1500 && f = \frac{12{,}000{,}000}{\frac{Ld}{bt}} \end{aligned} \qquad (10)$$

The allowable unit stresses in the above formulas are given in pounds per square inch. These formulas are plotted in Fig. 16.

Comparison with the AREA and AASHO formula in Eq. 7 was possible for only the shorter beams permitted under these specifications. Because the L/b parameter did not closely predict the nominal bending strength of the simply supported beams, the factors of safety with respect to these formulas were subject to considerable variation with the margin of safety in general being less for the more slender beams.

In the case of the simply supported beams, the formulas based on the Ld/bt parameter gave a fairly uniform factor of safety with respect to the proposed formulas, Eqs. 9 and 10, but with respect to the AISC formula, Eq. 8, only in the case of the more slender beams. These latter beams buckled in the elastic range. For the beams which buckled in the inelastic range, the factor of safety was dependent upon the yield point of the material and was as low as 1.19, as Table IV shows. If these beams had had the minimum yield point of 33,000 psi specified for structural carbon steel, the factor of safety for these beams would have been even lower with respect to the AISC specifications (Eq. 8). The cutoff curves used by the two proposed formulas (Eqs. 9 and 10) provided a much more adequate margin of safety for the shorter beams. In fact they afforded a factor of safety with few exceptions in excess of 1.50.

The beams with the two types of very flexible semi-rigid connections all developed factors of safety in terms of simple beam bending strength much greater than that which is usually considered necessary in a member subject to possible buckling. For values of Ld/bt below 1100, these types of connections would assure an entirely adequate margin of safety for beams designed by Eq. 8. For larger slenderness ratios, it would be desirable for design specifications to permit heavier loads than those now allowed in order to take advantage of the increased strength supplied by this type of connection which is usually considered to have no moment-carrying capacity.

It might be well to point out some of the uncertainties that exist in the design of beams with values of Ld/bt between 600 and 1100. Factors which have a significant effect upon the length of beams in this range of slenderness are,

1) The actual yield point of the material in the beam.
2) The initial misalignment of the beam.

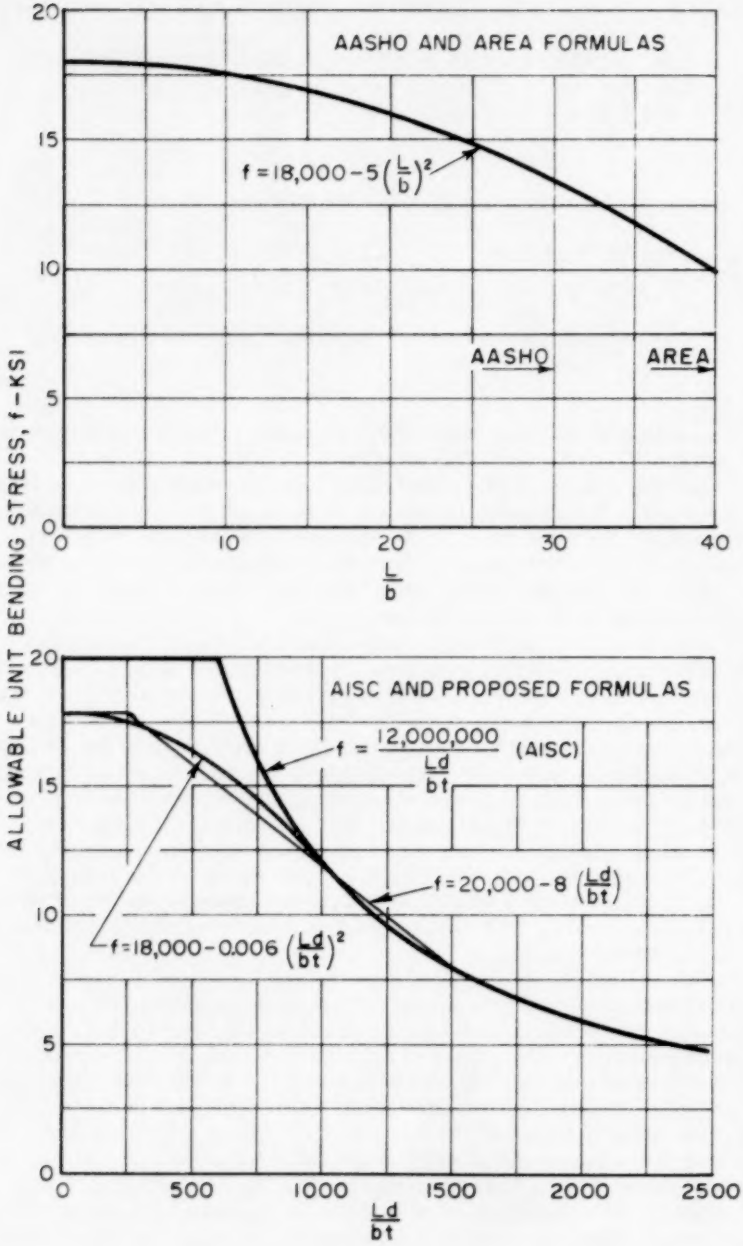

Fig. 16. Allowable Unit Bending Stresses by Present and Proposed Design Formulas.

3) The very adverse effect of failure to provide adequate lateral support of the span end cross-sections to secure them from rotating or tipping. Tests by Cyr and Hechtman[11] showed that short beams without any such support except that given by the beam seat might not even sustain the design load for a laterally unsupported simple beam.

CONCLUSIONS

The principal findings and conclusions of this investigation may be summarized as follows:

1) The test beams buckled in the elastic range when the value of Ld/bt was greater than about 1100 for the simply supported beams and those with web angle connections and greater than about 1300 for those with top and seat angle connections. Below these slenderness ratios, the beams buckled in the inelastic range.
2) When the beams failed in the elastic range, correlation between the experimental and theoretical values of the buckling load and nominal bending strength was obtained. Either the deVries (Eq. 1) or the Winter-Schrader (Eq. 6) formula accurately predicted the critical buckling loads.
3) When the beams failed in the inelastic range and the yield point of the steel and the shape of the stress-strain curve became factors, the actual bending strength of the beams was less than the theoretical and could be adequately represented by a cutoff curve similar to that which has already been experimentally established for intermediate length columns.
4) While the slenderness ratio Ld/bt became less definitive in predicting the nominal bending strength of the test beams as its value decreased, it was shown to be entirely adequate as a parameter for the design of rolled beams and is strongly recommended therefor.
5) The test beams developed an adequate factor of safety with respect to the design formula,

$$f = \frac{12{,}000{,}000}{\frac{Ld}{bt}}, \tag{8}$$

when a parabolic cutoff was used for values of Ld/bt less than 1000 or a diagonal straight-line cutoff for values less than 1500. The need for such a cutoff in the case of simply supported beams was shown by these tests.
6) The factors of safety of the test beams with respect to the design formula,

$$f = 18{,}000 - 5(L/b)^2, \tag{7}$$

were subject to considerable variation and in general decreased as the value of L/b increased. The factors of safety for the sections tested were adequate, but it is possible that other sections would not have given as favorable results. Moreover, this design formula is not applicable to slender beams which showed good load-carrying capacity in these tests.
7) For beams with values of Ld/bt between 600 and 1100, the two types of semi-rigid end connections provided an adequate factor of safety with

respect to Eq. 8 which assumes simple beam support. For greater slenderness ratios, it is desirable that design specifications take advantage of the increased strength supplied by this type of connection.

8) The buckling strength was found to be related to the initial misalignment and twist of the beams.
9) The Southwell method as adapted by Lundquist was found to be a useful tool for analyzing the data of lateral buckling tests of beams.

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BIBLIOGRAPHY

1. deVries, Karl, Strength of Beams as Determined by Lateral Buckling, Trans. ASCE, Vol. 112, 1947.
2. Timoshenko, S., Theory of Elastic Stability, McGraw-Hill, New York, 1936.
3. Winter, G., Discussion of Ref. 1, Trans. ASCE, Vol. 112, 1947.
4. American Institute of Steel Construction, Steel Construction Manual, 1946.
5. Schrader, R. K., Discussion of Lateral Stability of Unsymmetrical I-Beams and Trusses in Bending, Trans. ASCE, Vol. 108, 1943.
6. Wang, K. T., Notes on Lateral Buckling of I-Beams (Unpublished), University of Washington, 1950.
7. Van Eenam, Neil, Discussion of Ref. 1, Trans. ASCE, Vol. 112, 1947.
8. Julian, O. G., Discussion of Ref. 1, Trans. ASCE, Vol. 112, 1947.
9. Hechtman, R. A., and Johnston, B. G., Riveted Semi-Rigid Beam-to-Column Building Connections, AISC, New York, 1948.
10. Bleich, F., Buckling Strength of Metal Structures, McGraw-Hill, New York, 1952.
11. Cyr, N. C., and Hechtman, R. A., Lateral Buckling of Simply-Supported I-Beams, Trend in Engineering, University of Washington, July, 1950.

12. Southwell, R. V., On the Analysis of Experimental Observations in Problems of Elastic Stability, Proc. Roy. Soc. London, Vol. A135, 1932.

13. Donnell, L. H., On the Application of Southwell's Method for the Analysis of Buckling Tests, Contributions to the Mechanics of Solids Dedicated to Stephen Timoshenko, Macmillan, New York, 1938.

14. Lundquist, E. E., Generalized Analysis of Experimental Observations in Problems of Elastic Stability, Tech. Note No. 658, NACA, Washington, D.C., 1938.

15. Am. Rwy. Engrg. Assoc., Manual, Vol. II, Specifications for Steel Railway Bridges, 1953.

16. Am. Assoc. State Hwy. Off., Standard Specifications for Highway Bridges, Washington, D.C., 1953.

17. Research Committee E, Column Research Council, Report of Subcommittee, April 7, 1954.

NOTATION AND SYMBOLS

The notation and symbols used in this report may be defined as follows:

L	Span length. In these tests also the laterally unsupported length of compression flange of beam.
d	Depth of beam.
b	Width of beam flange.
t	Average thickness of beam flange.
E	Modulus of elasticity.
I_y	Moment of inertia of beam cross-section about y-axis; the minimum moment of inertia.
I_x	Moment of inertia of beam cross-section about x-axis; the maximum moment of inertia.
C	Torsional constant or stiffness of beam cross-section, M_t/θ.
a	Torsion-bending constant of beam cross-section, $\sqrt{EI_y d^2/4C}$
K	Torsional constant of beam cross-section, C/G.
P	Total load on beam.
P_{cr}	Critical load.
M_s	Simple beam moment of transverse loads on beam.
f_{cr}	Nominal bending strength or critical stress, $M_s c/I$.
f	Allowable unit bending stress.
k	Buckling factor.
β	Rotation of cross-section of beam with respect to vertical.
β_i	Initial angle of twist at midspan.

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